AUGMENTED TERNARY MAPS AND THEIR APPLICATIONS

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Motivated by the observation that when we have a ternary structure with some type of grading (decomposition compatible with the product) a complete knowledge of the grading set imply some structural theorems of the graded ternary structure, we consider an arbitrary non-empty set, which will play later the role of grading set, endow it with a new structure called *f-triple* and study it so as to obtain several decomposition results.

We begin the study by developing connections techniques among the elements of an f-triple $\mathfrak{A}$, so as to show that $\mathfrak{A}$ is the orthogonal (disjoint) union of a family of ideals $\{\mathfrak{I}_i : i \in I\}$. Then, we show that if furthermore $\mathfrak{A}$ is a division f-triple, then the decomposition obtained is through the family of its simple ideals.

Later, we consider some ternary structures with a grading. In particular, set-graded triple systems, supertriple systems admitting a multiplicative basis and set-graded algebraic pairs. If we treat the grading set as an adequate f-triple, all of the information previously obtained for f-triples will be translated into structural theorems of the initial graded ternary structure. These theorems follow the spirit of the the second Wedderburn theorem for associative algebras, see [1, pp. 137-139]).

Then, by applying the results obtained for f-triples, it is proved that any set-graded triple system $\mathcal{T}$ can be expressed as the orthogonal direct sum

$$\mathcal{T} = \bigoplus_j \mathfrak{I}_j$$

where any $\mathfrak{I}_j$ is a well described homogeneous-ideal of $\mathcal{T}$. If furthermore the grading is a weak-division grading, the homogeneous-simplicity of $\mathcal{T}$ is characterized and it is shown that $\mathcal{T}$ is the orthogonal direct sum of the family of its minimal homogeneous-ideals, each one being a homogeneous-simple triple system. Also, similar structure theorems to the ones for graded triple systems are given for the classes of arbitrary supertriple systems admitting a multiplicative basis and of set-graded algebraic pairs.

REFERENCES:

1. P.M. Cohn: *Basic Algebra: Groups, Rings, and Fields*, Springer-Verlag London (2003)