UNIVERSAL ENVELOPING OF LIE AND LEIBNIZ CROSSED MODULES

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Leibniz algebras, the non-antisymmetric analogue of Lie algebras, were first defined by Bloh [1] and later recovered by Loday in [5] when he handled periodicity phenomena in algebraic K-theory. Many authors have studied this structure and it has some interesting applications in Geometry and Physics ([2, 4, 7]).

Crossed modules of groups were described for the first time by Whitehead in the late 1940s [8] as an algebraic model for path-connected CW-spaces whose homotopy groups are trivial in dimensions greater than 2. From that moment, crossed modules of different algebraic objects, not only groups, have been considered, either as tools or as algebraic structures in their own right. Crossed modules of Lie algebras, hereafter called as Lie crossed modules, and crossed modules of Leibniz algebras are well known, at least as analogues of crossed modules of groups in another categories. Lie crossed modules have been investigated by various authors. Namely, in [3] Kassel and Loday use Lie crossed modules as computational tools in order to give an interpretation of the third relative Chevalley-Eilenberg cohomology of Lie algebras, and in [6] crossed modules of Leibniz algebras were defined in order to study cohomology.

The aim of this talk is to complete the task of building the universal enveloping crossed modules of a Lie crossed module and of a Leibniz crossed module.

On the one hand we construct a pair of adjoint functors between the categories of crossed modules of Lie and associative algebras, which extends the classical one between the categories of Lie and associative algebras. This result is used to establish an equivalence of categories of modules over a Lie crossed module and its universal enveloping crossed module.

On the other hand the universal enveloping algebra functor between Leibniz and associative algebras defined by Loday and Pirashvili is extended to crossed modules. We prove that the universal enveloping crossed module of algebras of a crossed module of Leibniz algebras is its natural generalization. Then we construct an isomorphism between the category of representations of a Leibniz crossed module and the category of left modules over its universal enveloping crossed module of algebras. Our approach is particularly interesting since the actor in the category of Leibniz crossed modules does not exist in general, so the technique used in the proof for the Lie case cannot be applied. Finally we move on to the framework of the Loday-Pirashvili category $\mathcal{LM}$ in order to comprehend this universal enveloping crossed module in terms of the Lie crossed modules case.

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