

# NILPOTENT ASSOCIATIVE ALGEBRAS AND COCLASS THEORY

**Tobias Moede**

*TU Braunschweig, Braunschweig, Germany*

*t.moede@tu-bs.de*

The coclass of a finite  $p$ -group of order  $p^n$  and class  $c$  is defined as  $n - c$ . In 1980 Leedham-Green & Newman suggested to use coclass as the primary invariant in a possible classification of finite  $p$ -groups. They proved first results and proposed five very detailed conjectures, known as Conjectures A-E. These conjectures became a driving force for the first years of coclass theory. By now all these conjectures have become theorems, but there are still challenging open problems in the area.

We have developed a coclass theory for nilpotent associative algebras over fields. A central tool in our investigation are so-called coclass graphs associated with the nilpotent associative  $\mathbb{F}$ -algebras of a fixed coclass. We have developed and implemented an algorithm to construct finite parts of these graphs over finite fields. Using the experimental evidence obtained in this way, we prove several structural results for the associated coclass graphs. This yields results in the flavour of the coclass theorems for finite  $p$ -groups.

The most striking observation in the experimental data is that for finite fields all of these graphs seem to exhibit a periodic pattern. It is ongoing research to prove this periodicity. Furthermore, we want to exploit this periodicity in order to describe the infinitely many nilpotent associative  $\mathbb{F}$ -algebras of a fixed coclass by finitely many parameterised presentations.