LEIBNIZ ALGEBRAS CONSTRUCTED BY REPRESENTATIONS OF GENERAL DIAMOND LIE ALGEBRAS

Karimjanov Ikboljon
University of Santiago de Compostela
iqboli@gmail.com

The Ado's theorem in Lie Theory states that every finite-dimensional complex Lie algebra can be represented as a matrix Lie algebra, formed by matrices. However, that result does not specify which is the minimal order of the matrices involved in such representations. In [1], the value of the minimal order of the matrices for abelian Lie algebras and Heisenberg algebras $\mathfrak{h}_m$, defined on a $(2m+1)$-dimensional vector space with basis $X_1, \ldots, X_m, Y_1, \ldots Y_m, Z$, and brackets $[X_i, Y_i] = Z$, is found. For abelian Lie algebras of dimension n the minimal order is $\lceil 2\sqrt{n-1} \rceil$.

The real general Diamond Lie algebra $\mathfrak{D}_m$ is a $(2m+2)$-dimensional Lie algebra with basis $\{J, P_1, P_2, \ldots, P_m, Q_1, Q_2, \ldots, Q_m, T\}$ and non-zero relations

$$[J, P_k] = Q_k, \qquad [J, Q_k] = -P_k, \qquad [P_k, Q_k] = T, \qquad 1 \leq k \leq m.$$

The complexification (for which we shall keep the same symbol $\mathfrak{D}_m(\mathbb{C})$) of the Diamond Lie algebra is $\mathfrak{D}_m \otimes_{\mathbb{R}} \mathbb{C}$, and it shows the following (complex) basis:

$$P_k^+ = P_k - iQ_k, \qquad Q_k^- = P_k + iQ_k, \qquad T, \qquad J, \qquad 1 \leq k \leq m,$$

where i is the imaginary unit, and whose nonzero commutators are

$$[J, P_k^+] = iP_k^+, \qquad [J, Q_k^-] = -iQ_k^-, \qquad [P_k^+, Q_k^-] = 2iT, \qquad 1 \leq k \leq m.$$

In this work we constructed a minimal faithful representation of the $(2m+2)$-dimensional complex general Diamond Lie algebra, $\mathfrak{D}_m(\mathbb{C})$, which is isomorphic to a subalgebra of the special linear Lie algebra $\mathfrak{sl}(m+2, \mathbb{C})$. We also constructed a faithful representation of the general Diamond Lie algebra $\mathfrak{D}_m$ which is isomorphic to a subalgebra of the special symplectic Lie algebra $\mathfrak{sp}(2m+2, \mathbb{R})$. Furthermore, we describe Leibniz algebras with corresponding $(2m+2)$-dimensional general Diamond Lie algebra $\mathfrak{D}_m$ and ideal generated by the squares of elements giving rise to a faithful representation of $\mathfrak{D}_m$.

REFERENCES:

1. Burde D. *Leibniz On a refinement of Ado's theorem*, Arch Math (Basel), **70(2)** (1998), 118-127.