LEIBNIZ ALGEBRAS WHOSE ASSOCIATED LIE ALGEBRA IS A WITT ALGEBRA[1]

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Leibniz algebra is a non-associative algebra with a bilinear product satisfying the so-called Leibniz identity. Each non-Lie Leibniz algebra L contains a non-trivial ideal (later denoted by I and usually called Leibniz kernel), which is generated by the squares of the elements of the algebra L, i.e. $I = \langle\{[x,x] \mid x \in L\}\rangle$. Moreover, it is easy to see that this ideal belongs to the right annihilator of L, that is $[L, I] = 0$. For a Leibniz algebra, L we consider the natural homomorphism φ into the quotient Lie algebra L/I, which is called *corresponding Lie algebra* to Leibniz algebra L (in some papers it is called *liezation* of L).

The map $I \times L/I \longrightarrow I$ defined as $(v, \overline{x}) \longrightarrow [v, x]$, $v \in I$, $x \in L$ endows I with a structure of a right L/I-module (it is well-defined due to I being in a right annihilator) knwon as the *hemisemidirect product of* L/I with I (see [3]).

Denote by $Q(L) = L/I \oplus I$, then the operation $(-,-)$ defines Leibniz algebra structure on $Q(L)$, where $(\overline{x}+v, \overline{y}+w) := [\overline{x}, \overline{y}] + [v, y]$, that is, $(\overline{x}, \overline{y}) = \overline{[x,y]}$, $(v, \overline{x}) = [v, x]$, $(\overline{x}, v) = 0$, $(v, w) = 0$, with $x, y \in L$, $v, w \in I$. In fact, this structure of Leibniz algebra is isomorphic to the initial one of L. Therefore, for a given Lie algebra G and a right G-module M, we can construct a Leibniz algebra $L = G \oplus M$ by the above construction.

One of the approaches in the investigation on Leibniz algebras is the description of these algebras such that their corresponding Lie algebras are a given Lie algebra (see [1,2]).

Using the construction of Leibniz algebras described above, we describe in this paper the infinite-dimensional Leibniz algebra associated to the Witt algebra. Let G be the Witt algebra with $\{d_i \mid i \in \mathbf{Z}\}$ a basis. Let $V(\alpha, b) = \{v(n) \mid n \in \mathbb{Z}\}$ be a G-module with the action given by a representation of Witt algebra [9]:

$$[v(n), d_m] = (\alpha + n + bm)v(n+m), \quad n \in \mathbb{Z}, \alpha, b \in \mathbb{C},\ \alpha \neq 0$$

Then, we construct the Leibniz algebra $L = G \oplus V$ as follows, we assume that the ideal $I = V$ and $L/I = G$. So we have the products

$$[V, G] \text{ defined as above} \quad [G, V] = [V, V] = 0 \quad \text{and} \quad [G, G] \subseteq G + V.$$

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