ON TERNARY ALGEBRAS AND A (NEW) TERNARY GENERALIZATION OF JORDAN ALGEBRAS

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This talk is devoted to n-ary algebras, which are generalized versions of certain classes of algebras by considering n-ary multiplications.

Perhaps the most remarkable example of n-ary algebras is the notion of *Filippov algebra* [1] (also known as n-Lie algebras) which is every algebra endowed with an anticommutative n-ary multiplication $[., ..., .]$ satisfying a *generalized Jacobi identity*:

$$[[x_1, ..., x_n], y_2, ..., y_n] = \sum_{i=1}^{n} [x_1, ..., [x_i, y_2, ..., y_n], ..., x_n]. \quad (1)$$

Since then, much has been done either by studying its structure, either by introducing some generalizations (*e.g.*, Filippov superalgebras).

In [3] Pozhidaev proposed the following generalization of Maltev algebras: an *n-ary Maltsev algebra* is an n-ary algebra equipped with an anticommutative n-ary multiplication $[., ..., .]$ satisfying a *generalized Maltsev identity*. A particular emphasis will be given to M_8, a certain ternary 8-dimensional Maltsev algebra defined on a composition algebra.

Recently [2], an n-ary generalization of Jordan algebras was proposed (different, in the ternary case, from Jordan triple systems and other generalizations). Let $\mathcal{A}$ be an n-ary algebra with a multilinear multiplication $[\![., \ldots, .]\!] : \times^n \mathbb{V} \to \mathbb{V}$, ($\mathbb{V}$ is the underlying vector space). $\mathcal{A}$ is said to be an *n-ary Jordan algebra* if $[\![., \ldots, .]\!]$ is totally commutative and satisfies $[R_x, R_y] \in Der(\mathcal{A})$. The first examples appear by considering some ternary algebras defined on the direct sum of a field and a vector space, equipping it with a product depending on three given forms. Working on a particular case restricted to a vector space, it is obtained a new example of simple ternary Jordan algebra, TJ_n, such that $Der(TJ_n) = Inder(TJ_n) = so(n)$. Further, two non-isomorphic symmetrized matrix ternary Jordan algebras are defined. Defining a certain ternary product on the algebras obtained by the Cayley-Dickson process, a 4-dimensional non-commutative ternary Jordan algebra over the generalized quaternions is built. An analog of the TKK-construction to the case of ternary algebras gives also new examples.

Other n-ary algebras will be mentioned, as well as relations among these.

REFERENCES:

1. V. T. Filippov, *n-Lie algebras*, Sib. Math. J., **26** (1985), 6, 126–140.

2. I. Kaygorodov, A. P. Pojidaev and P. Saraiva, *On a ternary generalization of Jordan algebras*, arXiv:1709.06826 (2017).

3. A. P. Pozhidaev, *n-ary Mal'tsev algebras*, Algebra and Logic, **40** (2001), 3, 309–329.